# Enthalpy and Heat Capacity of Steam-Helium and Steam-Oxygen Mixtures 

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In making design calculations for high pressure systems, the practicing engineer is often concemed with the transfer of heat. In many instances heat effects must be evaluated in high pressure flow systems which involve mixtures of jases. Although thermodynamic property charts or tables are available for some pure gases, very few thermodynamic properties have been established for gaseous mixtures at elevated pressures.

Enthalpy and heat capacity are the two thermodynamic properties most frequently used in engineering calculations. To calculate rigorously the effect of pressure on the enthalpy and heat capacity of a gaseous mixture or a pure component requires very accurate $P$-V-T data. Because such data are often unavailable, approximate methods must be used.
The purpose of this investigation was to estimate the enthalpy and heat capacity of steam-helium and steamoxygen mixtures at elevated temperatures and pressures.
The authors have presented $P-V-T$ data for steam-helium and steam-oxygen mixtures ( 6,7 ). Unfortunately, the $P-V-T$ behavior of these mixtures was not sufficiently established to permit rigorous calculation of the thermodynamic properties over the entire range investigated.
Sufficient $P-V-T$ data were available for a rigorous determination of the effect of pressure on the enthalpy of the steam-helium mixture at $650^{\circ} \mathrm{F}$. This calculation was made and the results are presented herein.

In the analysis of the $P-V-T$ data for these two mixtures, the authors showed that the $P-V-T$ behavior of these mixtures could be reliably predicted by use of Dalton's law and $P-V-T$ data on pure components. The steam-helium $P-V-T$ behavior was predicted to within $\pm 1.24 \%$; the steam-oxygen $P-V-T$ behavior, to within $-0.63 \%$. The success of Dalton's law in predicting these $P-V-T$ histories seems to indicate that each component acts independently in these mixtures. Thus, the enthalpies and heat capacities may also be calculated from the thermodynamic properties of the pure components at their respective partial pressures for the temperature in question.

To test the validity of the use of Dalton's law in calculating the thermodynamic properties of these mixtures, the effect of pressure on the enthalpy of the steam-helium mixture at $650^{\circ} \mathrm{F}$. was calculated by this technique. From a comparison of the results of this calculation to enthalpy change predicted from the $P-V-T$ data for this mixture it was concluded that Dalton's law and constituent thermodynamic properties may be used to predict the thermodynamic properties of these mixtures. Using this technique the enthalpy and heat capacity of these mixtures were calculated.
Presented herein are the enthalpies and heat capacities of a 70.54 mole $\%$ steam- 29.46 mole $\%$ helium mixture and a 71.07 mole \% steam- 28.93 mole \% oxygen mixture. These

[^0]properties are presented as a function of temperature over the range $550^{\circ}$ to $950^{\circ} \mathrm{F}$. for isobars from 1000 to 2000 pounds per square inch absolute.

## CALCULATIONS

Effect of Pressure on Enthalpy of Steam-Helium Mixture Using Mixture P.V-T Data. A complete set of thermodynamic properties for a single-phase system may be calculated from the $P-V-T$ data and heat capacity data at one pressure. However, $P-V-T$ data must be very accurate, to make this computation practical. For the mixture in question the $P-V-T$ data were not accurate enough for the calculation of thermodynamic properties over the entire range investigated, but it was deemed feasible to calculate the change of enthalpy with pressure at one temperature $\left(650^{\circ} \mathrm{F}\right.$.).

The basic thermodynamic relation used was:

$$
\begin{align*}
\Delta H_{T} & =\Delta E-\Delta P V \\
\Delta H_{T} & =J \int_{V_{1}}^{V_{2}}\left[T\left(\frac{\partial P}{\partial T}\right)_{V}-P\right] d V+J\left(P_{2} V_{2}-P_{1} V_{1}\right) \tag{1}
\end{align*}
$$

To evaluate each term in Equation 1 as accurately as possible, empirical equations were fitted to the experimental $P-V-T$ isochores (6) using linear or curve-linear regression (3). The equations developed were as follows, where $P=$ pressure in pounds per square inch absolute and $t=$ temperature in ${ }^{\circ} \mathrm{C}$.

Isochore 1. Molal volume, $12.75 \mathrm{cu} . \mathrm{ft} . / \mathrm{lb}$. mole
$\Gamma=854.8+1.526(t-331.4)$. Temperature range, $264^{\circ}$ to $412^{\circ} \mathrm{C}$.
Isochore 2. Molal volume, $10.17 \mathrm{cu} . \mathrm{ft} . / \mathrm{lb}$. mole
$P=1100+2.031(t-352.5)$. Temperature range, $286^{\circ}$ to $399^{\circ} \mathrm{C}$.
Isochore 3. Molal volume, $8.532 \mathrm{cu} . \mathrm{ft} / \mathrm{lb}$. mole
$P=1318+2.736(t-345.9)$. Temperature range, $301^{\circ}$ to $421^{\circ} \mathrm{C}$.
Isochore 4. Molal volume, $6.916 \mathrm{cu} . \mathrm{ft} . / \mathrm{lb}$. mole
$P=4.446 \times 10^{3}+6.096 \times 10 t-2.625 \times 10^{-1} t^{2}+5.412 \times 10^{-4} t^{3}-$ $4.238 \times 10^{-7} t^{4}$. Temperature range, $291^{\circ}$ to $386^{\circ} \mathrm{C}$.
Isochore 5. Molal volume, $6.158 \mathrm{cu} . \mathrm{ft} / \mathrm{lb}$. mole
$P=-1.558 \times 10^{4}+1.829 \times 10^{2} t-7.548 \times 10^{-1} t^{2}+1.419 \times 10^{-9} t^{3}-$ $1.003 \times 10^{-6} t^{4}$. Temperature range, $318^{\circ}$ to $410^{\circ} \mathrm{C}$.
Isochore 6. Molal volume, $5.294 \mathrm{cu} . \mathrm{ft}$. $/ \mathrm{lb}$.mole
$P=1.911 \times 10^{3}-1.811 \times 10 t+9.332 \times 10^{-2} t^{2}-1.168 \times 10^{-4} t^{3}$. Temperature range, $285^{\circ}$ to $355^{\circ} \mathrm{C}$.
Isochore 7. Molal volume, $4.579 \mathrm{cu} . \mathrm{ft} / \mathrm{lb}$.mole
$P=3.998 \times 10^{3}-6.208 \times 10 t+3.917 \times 10^{-1} t^{2}-9.272 \times 10^{-4} t^{9}+$ $7.830 \times 10^{-7} t^{4}$. Temperature range, $310^{\circ}$ to $376^{\circ} \mathrm{C}$.
The above empirical equations represent the experimental $P-V-T$ data of the steam-helium mixture to within the experimental precision of the investigation ( $\pm 2$ pounds per square inch). These equations were used to calculate the terms in the basic thermodynamic equation cited above. By algebraically solving for the $\frac{(\partial P)}{(\partial T)_{V}}$, the inherent error in the derivative was minimized. Tabulated in Table I are
[Temperature $343.33^{\circ} \mathrm{C}\left(650^{\circ} \mathrm{F}.\right)$ ]

| Isochore No. | Molal Volume, $\mathrm{Cu} . \mathrm{Ft} / \mathrm{Lb}$. Mole | Pressure, <br> Lb./Sq. Inch Abs. | $\begin{gathered} \frac{\partial P}{\partial T_{V}^{\prime}}, \\ \text { Lb. } / \text { Sq. } \text { Inch Abs. } /{ }^{\circ} \mathrm{K} . \end{gathered}$ | $T \frac{\partial P}{\partial T_{V}}$ | $T \frac{\partial P}{\partial T_{V}}-P$ | $P V$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 12.75 | 872 | 1.526 | 941 | 69 | 11,120 |
| 2 | 10.17 | 1082 | 2.031 | 1252 | 170 | 11,000 |
| 3 | 8.432 | 1311 | 2.736 | 1687 | 376 | 11,050 |
| 4 | 6.916 | 1552 | 3.500 | 2158 | 606 | 10,730 |
| 5 | 6.158 | 1733 | 4.050 | 2497 | 764 | 10,410 |
| 6 | 5.294 | 1966 | 4.660 | 2873 | 907 | 10,410 |
| 7 | 4.579 | 2219 | 5.770 | 3557 | 1338 | 10,160 |


| $\begin{gathered} V, \\ \mathrm{Cu} . \mathrm{Ft}_{\mathrm{t}} \end{gathered}$ | $\begin{gathered} P, \\ \text { Lb./Sq. } \\ \text { Inch Abs. } \end{gathered}$ | Area from$\begin{aligned} & T \frac{\partial P}{\partial T_{V}}-P \text { Vs. V, } \\ & \text { Lb./Sq. Inch Abs.- } \\ & \text { Cu. Ft. } \\ & \text { Lb. Mole } \end{aligned}$ | ct of Pressure on En <br> tum plane. $872 \mathrm{lb} . / \mathrm{sq}$ | alpy of Steam-Hell nch abs. and 650 | Mixture | $\begin{aligned} & \triangle P V, \\ & \text { B.t.u. } \end{aligned}$ | $\begin{aligned} & \Delta H_{T}, \\ & \text { B. t. u. } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Cumulative Area, Lb./Sq. Inch Abs.Cu. Ft. | Internal <br> Energy Change, <br> B.t.u./Lb. Mole | $\Delta P V$,Lb. $/$ Sq Inch Abs -Cu. Ft.Lb. Mole |  |  |
| Lb. Mole |  |  | Lb. Mole |  |  | Lb. Mole | Lb. Mole |
| 12.75 | 872 | $\ldots$ | . | $\ldots$ | . . | . | $\ldots$ |
| 10.17 | 1082 | -273 | -273 | -51 | -120 | -22 | -73 |
| 8.432 | 1311 | -458 | -731 | -135 | -50 | -9 | -144 |
| 6.916 | 1552 | -745 | -1477 | -273 | -390 | -72 | -345 |
| 6.158 | 1733 | -515 | -1992 | -368 | -450 | -83 | -451 |
| 5.294 | 1966 | -731 | -2723 | -504 | -710 | -131 | -635 |
| 4.579 | 2219 | -737 | -3460 | -640 | -960 | -178 | -818 |

calculated parameters which were used in completing this computation. In Table II the results of graphically integrating equation one are tabulated, with the change of enthalpy with pressure.

Enthalpy by Dalton's Law and Constituent Thermodynamic Data. Sample calculation shown for 70.54 mole $\%$ steam29.46 mole $\%$ helium mixture at $1000 \mathrm{lb} . / \mathrm{sq}$. inch abs. and temperature of $650^{\circ} \mathrm{F}$. Basis. One mole of mixture.

| 1. Assume pastial pressure of steam | 682 1b. /sq inch abs. |
| :---: | :---: |
| Specific volume steam at $682 \mathrm{lb} . / \mathrm{sq}$ inch abs. and $650^{\circ} \mathrm{F}$. (5) | $0.8806 \mathrm{cu} . \mathrm{ft}$ |
| Mixture molal volume |  |
| (0.8806) (18.016) (0.7054) | 11.19 cu ft . |
| Helium molal volume |  |
| 11.19 | 37.9 cu. ft |
| 0.2946 |  |
| Partial pressure of helium at $37.9 \mathrm{cu} . \mathrm{ft} / \mathrm{lb}$. mole and $650^{\circ} \mathrm{F}$. (1) | 317.4 lb. /sq inch abs. |
| Total pressure $=682+317=$ | $999 \mathrm{lb} . / \mathrm{sq}$. inch abs. |
| 2. Enthalpy of steam at $0.8806 \mathrm{cu} \mathrm{ft}_{\mathrm{o}} / \mathrm{lb}$. and $650^{\circ} \mathrm{F}$. (5) | $\begin{aligned} & 1315.4 \text { B.t.u./ } \\ & \text { 1b. } \end{aligned}$ |
| Enthalpy of steam for 1 lb . mole of mixture $(1315.4)(18.016)(0.7054)=$ | 16,717 B.tu. |
| 3. Calculation of helium enthalpy at $317 \mathrm{lb} . / \mathrm{sq}$. inch abs. and $650^{\circ} \mathrm{F}$. |  |
| Datum plane chosen as $32^{\circ} \mathrm{F}$. and $4.4 \mathrm{lb} . / \mathrm{sq}$ Inch abs. |  |
| Mean heat capacity of helium between $32^{\circ}$ and $650^{\circ} \mathrm{F}$. (4) | 4. 965 B.t.u./ 1b. mole ${ }^{\circ} \mathrm{F}$. |
| Helium enthalpy at $650^{\circ} \mathrm{F}$. and $4.4 \mathrm{lb} /$ sq. inch abs. |  |
| $(4.965)(650-32)(0.2946)=$ | 904 B.t.u./lb. mole mixture |

$$
\begin{aligned}
& \text { Helium enthalpy variation with pressure. } \\
& \left.\begin{array}{rl}
\frac{\partial H}{\partial P} & =\left[V-T\left(\frac{\partial V}{\partial T}\right)_{P}\right] \\
\Delta H & \left.=\begin{array}{r}
317 \\
4.4
\end{array}\right\} V d P-1100 \\
317 \\
4.4
\end{array}\right)\left(\frac{\partial V}{\partial T}\right)_{P} d P
\end{aligned}
$$

Both of the above integrals were evaluated using the Beattle-Bridgeman equation of state (2)
$\Delta H=3.59$ B.t.u. $/ \mathrm{b}$. mole helium
Enthalpy change for 0.2946 lb . mole helium $(3.59)(0.2946)=$

Total enthalpy change for hellum $=904+$ $1.06=905$ B.t.u. $/ 1 \mathrm{~b}$. mole mixture
Assuming no heat of mixing the steam and hellum vapors.
Total enthalpy change for mixture $=$ enthalpy of steam + en-

|  | thalpy of helium |
| :---: | :---: |
| Above $32{ }^{\circ} \mathrm{F}$. liquid water | $=16,717+905$ |
| $4.4 \mathrm{lb} . / \mathrm{sq}$. inch abs. helium datum |  |
| plane | $=17,622$ B. t.u. per 1b. mole |

## DISCUSSION OF RESULTS

Enthalpy. The effect of pressure on the enthalpy of a steam-helium mixture is shown in Figure 1, where the change in enthal py with pressure at $650^{\circ} \mathrm{F}$. calculated from the steam-helium $P-V-T$ data is presented for a pressure range of 872 to 2219 pounds per square inch absolute. From the smooth curve through these data, the change of enthalpy with pressure was calculated for a datum plane of 1000 pounds per square inch absolute and $650^{\circ} \mathrm{F}$. In Table III the changes in enthalpy with pressure calculated from mixture $P-V-T$ data and Dalton's law and component enthalpy


Figure 1. Change of enthalphy with pressure at $650^{\circ} \mathrm{F}$.

- Steam-helium mixture based on $P-V-T$ data Datum plane $872 \mathrm{lb} . / \mathrm{sq}$. inch abs.
$\Delta$ Steam-helium mixture based on Dalton's law Datum plane $1000 \mathrm{lb} . / \mathrm{sq}$, inch abs.
... Smoothed steam-helium data based on $P-V-T$ data Datum plane $1000 \mathrm{lb} . / \mathrm{sq}$. inch abs.

Table III. Effect of Pressure on Enthalpy
of Steam-Helium Mixture
(Datum plane. $1000 \mathrm{lb} . /$ sq. inch abs. and $650^{\circ} \mathrm{F}$.)
Deviation,
$\Delta H$ Dalton's

data are compared. The maximum deviation between the $\Delta H$ 's calculated by the two methods was -59 B.t.u., and the mean deviation was -49 B.t.u. These $\Delta H$ 's yielded the same shaped curves when plotted against pressure in Figure 1. While there is a constant error between the two methods of calculation, it was concluded that the comparison justifies the use of Dalton's law and component enthalpy data, as a mean difference of -49 B.t.u. will not significantly affect the calculation of the total enthalpy of the mixture.
Thus, the enthalpies of the steam-helium and steam-oxygen mixture were calculated from Dalton's law and constituent thermodynamic properties. The sources of the oxygen thermodynamic data were:

1. $P \cdot V-T$ data were estimated from Beattie-Bridgeman equation (2). 2. Heat capacity of oxygen was taken from Keenan and Kaye (4).

The molal enthalpy of the 70.54 mole $\%$ steam- 29.46 mole $\%$ helium mixture and the molal enthalpy of the 71.07 $\mathrm{mole} \%$ steam- 28.93 mole $\%$ oxygen mixture are presented as a function of temperature in Figure 2. Enthalpies are presented in Table IV.
Heat Capacity. The heat capacity of these mixtures was first calculated by graphical differentiation of the enthalpytemperature isobars. As these enthalpy isobars had only slight curvature except as saturation was approached, it was difficult to differentiate these curves accurately. Thus, heat capacities were also calculated from Dalton's law and component heat capacity data.

Table IV. Molal Enthalpy Calculated Using Dalton's Law and Constituent Thermodynamic Properties

|  | Enthalpy, B.t.u./Lb. Mole of Mixture |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1000 | 1200 | 1400 | 1600 | 1800 | 2000 |
| Temp., | $1 \mathrm{~b} . / \mathrm{sq}$. | $1 \mathrm{~b} . / \mathrm{sq}$. | $1 \mathrm{~b} . / \mathrm{sq}$. | 1b./sq. | $1 \mathrm{~b} . / \mathrm{sq}$. | $1 \mathrm{~b} . / \mathrm{sq}$. |

${ }^{\circ} \mathrm{F}$. inch abs. inch abs. inch abs. inch abs. inch abs. inch abs.

## Steam-Helium Mixture

| 550 | 16,585 | 16,435 | -••* | -••• | -••* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 580 | 16,910 | 16,750 | 16,605 | 16,410 | 16,125 | 15,860 |
| 600 | 17,134 | 17,978 | 16,870 | 16,720 | 16,452 | 16,210 |
| 650 | 17,622 | 17,500 | 17,374 | 17,243 | 17,111 | 16,940 |
| 700 | 18,100 | 17,991 | 17,885 | 17,771 | 17,660 | 17,548 |
| 800 | 18,970 | 18,890 | 18,811 | 18,743 | 18,650 | 18,581 |
| 850 | 19,400 | 19,328 | 19,263 | 19,192 | 19,114 | 19,050 |
| 900 | 19,824 | 19,758 | 19,705 | 19,640 | 19,577 | 19,517 |
| 950 | 20,260 | 20,197 | 20,144 | 20,091 | 20,034 | 19,963 |
| Steam-Oxygen Mixture |  |  |  |  |  |  |
| 550 | 17,080 | 16,871 | 16,654 |  |  |  |
| 600 | 17,666 | 17,492 | 17,337 | 17,158 | 16,966 | 16,835 |
| 700 | 18,716 | 18,611 | 18,507 | 18,387 | 18,279 | 18,151 |
| 800 | 19,717 | 19,632 | 19,557 | 19,472 | 19,392 | 19,292 |
| 900 | 20,686 | 20,614 | 20,558 | 20,481 | 20,427 | 20,366 |




Figure 2. Enthalpy-temperature isobars
Basis Dalton's law
A. $1000 \mathrm{lb} . / \mathrm{sq}$. inch abs.
B. $1200 \mathrm{lb} . / \mathrm{sq}$. inch abs.
C. $1400 \mathrm{lb} . / \mathrm{sq}$. inch abs.
D. $1600 \mathrm{1b} . / \mathrm{sq}$. inch abs.
E. $1800 \mathrm{lb} . / \mathrm{sq}$. inch abs.
F. $2000 \mathrm{lb} . / \mathrm{sq}$. inch abs.

Upper. 70.54 mole $\%$ steam 29.46 mole $\%$ helium

Lower. 71.07 mole \% steam 28.93 mole \% oxygen

Table V. Molal Heat Capacity Calculated by Graphical Differentiotion
Heat Capacity, B.t.u. $/$ Lb. Mole, ${ }^{\circ}$ F.
Temp., ${ }^{\circ}$ F.

| Heat Capacity, B.t.u./Lb. Mole, ${ }^{\circ} \mathrm{F}$. |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| 1000 | 1200 | 1400 | 1600 | 1800 | 2000 |
| lb./sq. inch abs. | lb./sq. inch abs. | lb./sq. inch abs. | lb./sq. inch abs. | lb./sq. inch abs. | lb./sq. inch abs. |

506 (satn.)
520 (satn.)
532 (satn)
552 (satn.)
563 (satn)
572 (satn.)
600
650
700
750
800
850
900
11.8
$\ldots$
$\ldots$
$\ldots$
$\ldots$
10.1
9.2
8.9
8.4
8.2
8.2
8.2

Steam-Helium Mixture (Figure 2, upper)

|  |  |  |
| :---: | :---: | :---: |
| $\ldots$. | $\ldots$ | $\ldots$ |
| $\ldots \ldots$ | $\ldots$ | $\ldots$ |
| $\ldots$ | $\ldots$ | $\ldots$ |
| $\ldots$ | $\ldots$ | 14.5 |
| 11.0 | 11.6 | $\ldots$ |
| 9.4 | 9.8 | 13.2 |
| 9.1 | 9.4 | 10.4 |
| 8.9 | 9.1 | 9.7 |
| 8.6 | 8.8 | 9.6 |
| 8.6 | 8.8 | 9.0 |
| 8.4 | 8.6 | 9.0 |
|  |  | 8.7 |


| Steam-Oxygen Mixture (Figure 2, lower) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 498 (satr.) | 15.1 | . . | . $\cdot$ | $\ldots$ | . $\cdot$ |  |
| 517 (satn.) | ... | 15.6 | . | . . | ... | $\cdots$ |
| 534 (satr.) | $\ldots$ | ... | 16.3 | $\ldots$ | $\cdots$ | . . . |
| 549 (satr.) | $\cdots$ | . $\cdot$ | ... | 17.1 | . $\cdot$ | . . . |
| 560 (satn.) | $\ldots$ | $\cdots$ | $\cdots$ | ... | 18.3 | . . . |
| 572 (satr) | ... | . . | . . | . . | ... | 20.8 |
| 600 | 11.1 | 12.0 | 12.7 | 13.7 | 14.8 | 17.1 |
| 650 | 10.5 | 11.0 | 11.1 | 12.4 | 12.7 | 13.1 |
| 700 | 9.6 | 10.6 | 10.5 | 11.1 | 11.4 | 12.2 |
| 750 | 9.4 | 9.9 | 10.2 | 10.6 | 10.8 | 11.6 |
| 800 | 9.4 | 9.7 | 10.1 | 10.4 | 10.3 | 11.0 |
| 850 | 9.3 | 9.6 | 10.0 | 9.8 | 10.3 | 10.5 |
| 900 | 9.2 | 9.4 | 9.6 | 9.6 | 10.2 | 10.3 |




Figure 3. Heat capacity-temperature isobars
A. $1000 \mathrm{lb} . / \mathrm{sq}$. inch abs.
B. $1200 \mathrm{lb} . / \mathrm{sq}$. inch abs.
C. $1400 \mathrm{lb} . / \mathrm{sq}$. inch abs.
D. $1600 \mathrm{lb} . / \mathrm{sq}$. inch abs.
E. $1800 \mathrm{lb} . / \mathrm{sq}$. inch abs.
F. $2000 \mathrm{lb} . / \mathrm{sq}$. inch abs.

## Left. Steam-helium mixture

Basis. Graphical differentiation Figure 2, upper
Right. Steam-oxygen mixture
Basis. Graphical differentiation Figure 2, lower

Calculation of Mixture Molal Heat Capacity Using Dalton's Law. Sample calculation shown for 70.54 mole $\%$ steam- 29.46 mole \% helium mixture are total pressure of $1000 \mathrm{lb} . / \mathrm{sq}$ inch abs. and temperature of $600^{\circ} \mathrm{F}$.

## Partial pressure of steam

$680 \mathrm{lb} . / \mathrm{sq}$. inch abs.
Heat capacity of steam at $680 \mathrm{lb} . / \mathrm{sq}$. inch abs. and $600^{\circ} \mathrm{F}$. (5)

Partial heat capacity of mixture due to steam

$$
(0.691)(18.016)(0.7054)=
$$

Partial pressure of helium
Heat capacity of helium at $4.4 \mathrm{lb} . / \mathrm{sq}$. inch abs. and $600^{\circ} \mathrm{F}$. (4).
0.691 B.t.u. per 1b., ${ }^{\circ} \mathrm{F}$.
8.79 B.t.u.

1b. mole mix.
320 1b. /sq. inch abs.
4. 965 B.t.u.
$\overline{\text { 1b. mole }{ }^{\circ} \mathrm{F} \text {. }}$

Variation of helium heat capacity with pressure

$$
\begin{aligned}
& \frac{\partial C_{P}}{\partial P_{T}}=-T \frac{\partial^{2} V}{\partial T_{P}^{2}} \\
& \Delta C_{P}=-T \int_{4.4}^{320} \frac{\partial^{2} V}{\partial T_{P}^{2}} d P
\end{aligned}
$$

This integral was evaluated using the Beattie-Bridgeman equa-


Figure 4. Heat capacity-temperature lsobars
Basis. Dalton's law
A. $1000 \mathrm{lb} . / \mathrm{sq}$. inch abs.
B. $1200 \mathrm{lb} . / \mathrm{sq}$. inch abs. C. $1400 \mathrm{lb} . / \mathrm{sq}$. inch abs. D. $1600 \mathrm{lb} . / \mathrm{sq}$. inch abs. E. $1800 \mathrm{lb} . / \mathrm{sq}$. inch abs. F. $2000 \mathrm{lb} . / \mathrm{sq}$. inch abs.

Left. Steam-hellum mixture
Right. Steam-oxygen mixture

Table VII. Comparison of Molal Heat Capacities Calculated by Dalton's Law and Heat Capacities of Constituents with Molal Heat Capacity Calculated by Graphical Differentiotion


## NOMENCLATURE

$C_{p}=$ molal heat capacity, B.t.u./lb. mole
$E=$ internal energy, B.t.u./lb. mole
$H=$ enthalpy, B.tu/lb. mole
$P=$ pressure, $\mathrm{lb} . / \mathrm{sq}$. inch abs.
$T=a b s o l u t e$ temperature, ${ }^{\circ} \mathrm{K}$.
$t=$ temperature, ${ }^{\circ} \mathrm{C}$.
$V=$ molal volume $-\mathrm{cu} . \mathrm{ft} . / \mathrm{lb}$, mole
$J=$ proportionality factor

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# Vapor Pressure and Viscosity of Solutions in the Calcium Oxide-Phosphoric Oxide-Water System at $25^{\circ} \mathrm{C}$. 

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A diversity of interest in the properties of phosphoric acid and calcium phosphate solutions in industrial, agronomic, and biochemical fields has led to the publication of articles conceming various properties of such solutions (2-4, 6-10, 13,15 ). The vapor pressures and viscosities of solutions in the system calcium oxide-phosphoric oxide-water, as measured by usual methods, are reported here. Compositions of the solutions were bounded by the line phosphoric oxide-water and the solubility isotherms of anhydrous dicalcium phosphate and monocalcium phosphate monohydrate $(2,8)$. The solution compositions lay along lines of constant weight ratios of phosphoric oxide-calcium oxide, as shown in Figure 1.

## PREPARATION OF SOLUTIONS

The solutions were prepared from conductance water, recrystallized phosphoric acid hemihydrate, and monocalcium phosphate monohydrate. These reagents were prepared by methods described elsewhere (15), and their purity was comparable to those of the same reagents used for measurements of density, conductance, and pH (15). Stock solutions with the phosphoric oxide-calcium oxide weight ratios shown in Figure 1 were prepared by mixing weighed amounts of monocalcium phosphate monohydrate, water, and phosphoric acid of known concentration. The stock solutions are those represented by the points of highest concentration for each phosphoric oxide-calcium oxide ratio line in Figure 1. For individual measurements, more dilute solutions were prepared by adding water to these stock solutions. The composition of each solution was checked by measuring its density (15) and comparing the result with tables of the density-composition relation for solutions in the system (15).

## METHODS OF MEASUREMENT

Vapor Pressure. The vapor pressure measurements were made by a differential static method. Conductance water was the reference material. As the dimensions of the apparatus were unsuitable for measurement of the vapor pressure of water, a published value, 23.756 mm . of mercury (14), was used. The advantages of the differential method for measuring vapor pressures of aqueous salt solutions have been discussed (12). The apparatus, Figure 2, was a modification of one described by Gibson and Adams (12).


Figure 1. Composition of test solutions


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